Beam simulation of CERN SPS

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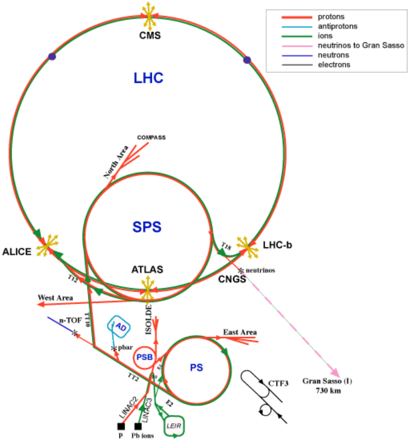
Outline

1. Motivation
2. Simulation results
   - Emittance growth due to noise
   - Emittance growth due to space-charge
   - Emittance growth due to tune modulation
3. Compact crab cavity model
4. Summary

SPS (Super Proton Synchrotron)

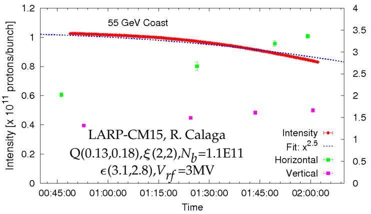
- Circumference: 6.9 km
- injection energy at 26 GeV/c
- protons for LHC at 450GeV/c
- protons for fixed target physics at 400 GeV/c
- protons for CNGS experiment at 400GeV/c



LHC: Large Hadron Collider
SPS: Super Proton Synchrotron
AD: Antiproton Decelerator
ISOLDE: Isotope Seperator OnLine DEvice
PSB: Proton Synchrotron Booster
PS: Proton Synchrotron
LINAC: LINear ACcelerator
LEIR: Low Energy Ion Ring
CNGS: Cern Neutrinos to Gran Sasso

Motivation

- During 2010 machine study in the SPS, large transverse emittance growth rate (ϵ_x=80%/h, ϵ_y=42%/h) has been observed at 55 GeV.
  - Dipole voltage ripple, space-charge, tune modulation, RF phase noise, chromaticity, IBS, ... contribute to the emittance growth.



- Machine studies on low transition energy in the SPS are done in 2011 to understand the source of emittance growth.
- We investigate the emittance growth in both nominal γ_T (22.90) and low γ_T (18.01).
- SPS is one of promising candidates for testing crab cavity for the HL-LHC.

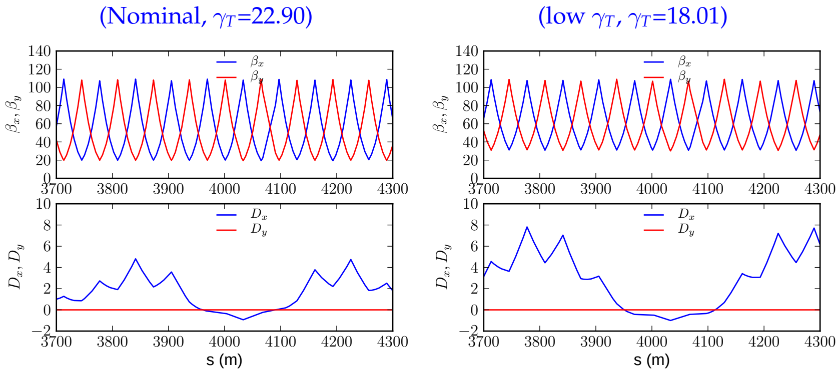
Low transition gamma (γ_T)

parameter	symbol	nominal	low γ_T
transition energy		22.90	18.01
transverse tune	(ν_x, ν_y)	(26.13, 26.18)	(20.13, 20.18)
natural chromaticity		(-32.68, -32.74)	(-22.79, -22.83)
sextupole strength	m^{-2}	(0.063, -0.150)	(0.045, -0.041)
max. beta	(β_x, β_y)	(111, 109)	(109, 109)
max. dispersion	(η_x, η_y)	(4.9, 0)	(8.1, 0.0)
beam energy	GeV	55	
beam intensity		1×10^{11}	
chromaticity	(ξ_x, ξ_y)	(0, 0)	
transverse emittance	mm-mrad	3.5	
long. emittance, 4σ	eV s	0.24	
rf voltage	MV	3	
particle distribution		Gaussian in (x, y, z)	

(Courtesy of I. Papaphilippou)

- Note) natural chromaticity and focusing sextupole strength of nominal optics are 40% larger than those of low γ_T.

Optics in nominal and low γ_T



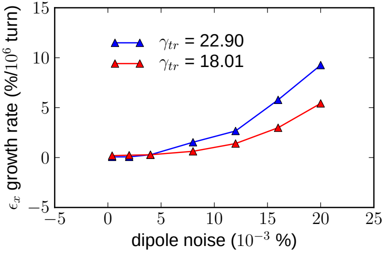
- Weaker focusing has the consequence of increasing beta functions and dispersions - both of which increase the beam size.
- β_{max} does not change much, but β_{min}=20m (nominal), β_{min}=34 (low γ_T).
- maximum dispersions 4.9 (nominal), 8.1 (low γ_T), but mimimum dispersion -0.91 (nominal), -0.98 (low γ_T).

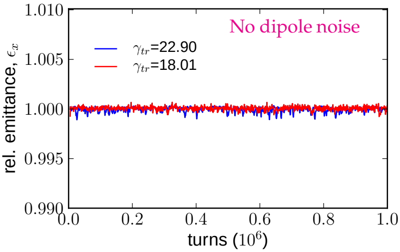
Beam-Beam Simulation (BBSIM) code

- 6D weak-strong tacking code.
- Linear transfer matrices btwn nonlinear elements + nonlinear kicks at the nonlinear elements (thin lens approximation: dipole, quadrupole, sextupole, mulitpole, etc.).
- Space charge: 2-D and 3-D Poisson solvers using (1) Conjugate Gradient and (2) FFT.
- Beam-beam force: (1) Gaussian beam profile and (2) Poisson solver with FFT.
- Multiple-slice model for finite bunch length effects.
- Lorentz boost to handle crossing angle collisions.
- Modules: crab crossing, wire and electron lens compensation, etc.
- Fully parallelized with MPI.
- Simulations agree well with measurements in the Tevatron, RHIC. Also applied to wire compensation in the SPS, LHC.
- Diagnostics: Beam loss, emittance growth, beam profiles, BTFs, dynamic aperture, tune footprints.

Emittance growth vs dipole noise

- Large emittance growth in the SPS (MD2010). Expect that dipole noise contributes to the emittance growth.
- Gaussian distribution with 10000 particles, 10^6 turns (23 seconds).
- Model: sextupole + dipole voltage ripple (white noise)
- Emittance growth is (2 times) less in low γ_T.
- Sextupole strength of nominal is 40% larger than low γ_T.
- Vertical and longitudinal emittance growth is insignificant.
- Voltage ripple of LHC after active filtering is 2.5×10^{-3}%.





2.5-D space-charge kicks

- Transverse electric field (fast 2D Poisson solver, $\vec{E} = -\nabla\phi$)

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\phi\,(x, y) = -\frac{1}{\epsilon_0}\rho\,(x, y)$$

- longitudinal electric field (ρ_L line density)

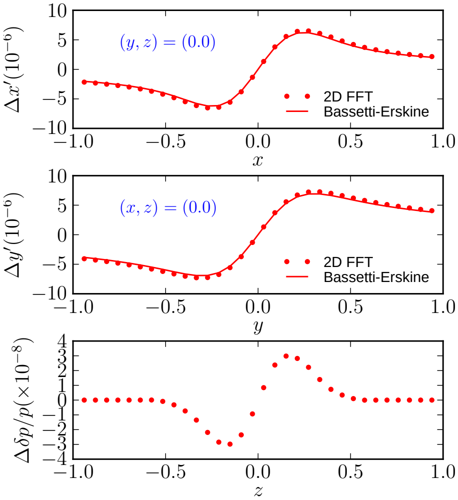
$$E_z = -\frac{g}{4\pi\epsilon_0\gamma^2}\frac{d}{dz}\rho_L\,(z)$$

- Space charge kicks:

$$\Delta\vec{r'} = \frac{qL}{m_0c^2\beta^2\gamma^3}\vec{E}\,(x, y)\,\frac{\rho_L(z)}{\rho_0},$$

$$\Delta\frac{\delta p}{p} = \frac{qL}{m_0c^2\beta^2\gamma}E_z\,(z)\,.$$

- Benchmark with 10,000 particles in SPS optics. Space-charge kicks obtained by Poisson solver and Bassetti-Erskine formula are well agreed.

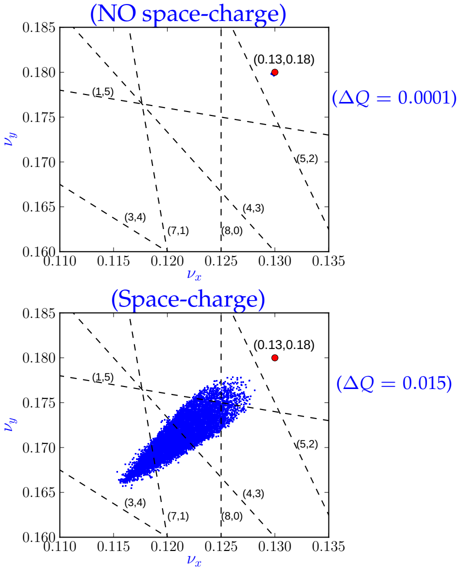


Tune footprint (space charge)

- Gaussian distribution in (x, y, z) with 10,000 particles.
- Apply space-charge kicks at quadrupole locations.
  - 18, 36, 72, and 208 kicks per turn are tested.
  - 72 kicks/turn is chosen.
- Tune shift for bunched beam due to space-charge

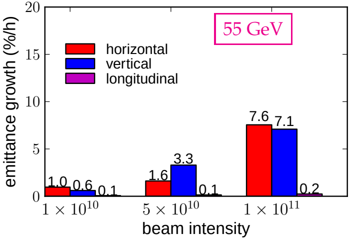
$$\Delta Q = -\frac{N_b r_p}{4\pi B \beta \gamma^2 \epsilon_N}$$

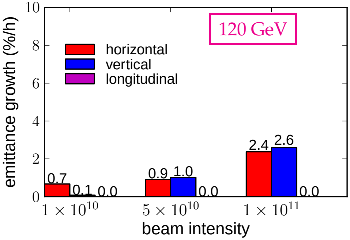
- Tune shift of particles with small betatron amplidue (55GeV, $N_b = 10^{11}$, ϵ_N=3.5μm, $\sigma_z = 0.18m$), $\Delta Q = 0.015$.
- 6-th, 7-th, and 8-th resonance lines are spanned.



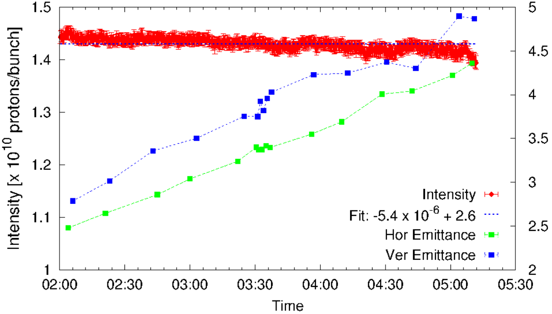
Emittance growth (space-charge)

- low γ_T (18.01) lattice.
- Gaussian distribution in (x, y, z) with 60,000 particles.
- No noises are added in the model.
- Space charge kicks at 72 locations.
- Space-charge induces 7%/hr emittance growth in both horizontal and vertical planes. 0.2%/hr emittance growth in longitudinal plane.
- γ_T (22.90) has the same growth.
- Space-charge is expected to have a small contribution to observed emittance growth in MD2010 .
- (Emittance growth due to space-charge + dipole noise equals the sum of emittance growth due to space-charge and dipole noise respectivley.)





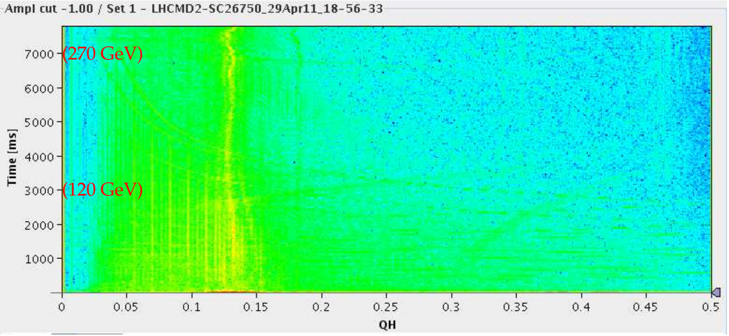
SPS machine study (120GeV, May 25, 2011)



(Coutesy of R. Calaga)

- Beam energy is 120GeV. Low transition energy (18.01).
- Tunes are nominal (0.13, 0.18). Chromaticity is close to 0.5 units.
- Observed about 25%/hr emittance growth in both horizontal and vertical planes.
- Tunes are scanned upto 0.35, but there is no effect on the slope of the emittance.

Tune modulation



(Coutesy of R. Calaga)

- Horizontal tune spectra during the energy ramp 26-450 GeV. (Vertical tune spectra is similar to horizontal one.)
- Beam energy 120GeV is around 3084 ms. Horizontal nominal tune is $\nu_x = 0.13$.
- One can see a lot of harmonics with relative amplitudes below 4000 ms. In order to see the effect of tune modulation, an experiment was done at 270 GeV in July, 2011. We see about 20%/hr emittance growth as well.

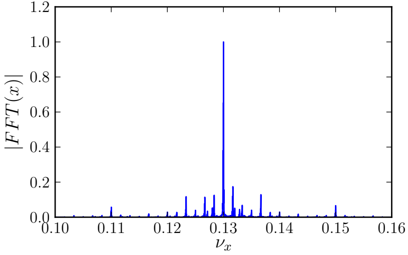
Tune modulation

k	Ω_k	$\epsilon_k \left(\times 10^{-4}\right)$
1	$2\pi/867.5$	1.000
2	$2\Omega_1$	0.218
3	$3\Omega_1$	0.708
4	$6\Omega_1$	0.254
5	$7\Omega_1$	0.100
6	$10\Omega_1$	0.078
7	$12\Omega_1$	0.218

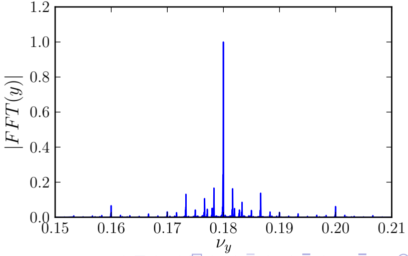
(M. Giovannozzi, Phy. Rev. E, 1998)

- Considered one main frequency Ω_1 and six harmonics with relative amplitudes.
- Tune modulation due to the observed ripple in the quadrupoles of the SPS.

(Horizontal tune spectra)



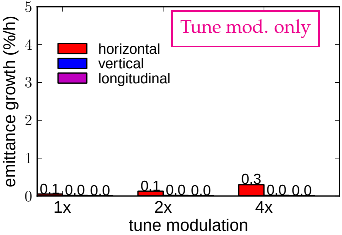
(Vertical tune spectra)

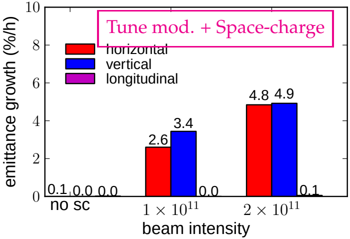


Emittance growth (tune modulation)

- low γ_T (18.01) lattice.
- 120 GeV and 270 GeV beam energy.
- 7 harmonics are included in the model (previous slide).
- No emittance growth due to tune modulation in both 120 GeV and 270 GeV.
- 1x - amplitude of tune modulation as shown in previous table, 2x - double amplidude of 1x.

- Space charge kicks at 72 locations.
- Emittance growth due to space-charge is about the same as that due to space-charge + tune modulation.



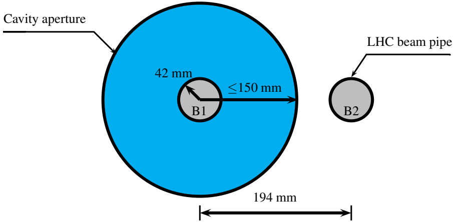


Summary of SPS simulation

- Emittance growth rate studies for nominal and low γ_T optics. Noises affect low γ_T optics (two times) less than nominal optics.
- Space-charge increases emittance in both nominal and low γ_T optics. The emittance growth is approximately 7%/hr in both horizontal and vertical planes at 55 GeV, 2.5%/hr at 120 GeV, and $\simeq$0%/hr at 270 GeV.
- Tune modulation has no effect on emittance growth at nominal tune (0.13, 0.18). However, the spectral lines in the SPS now may be different.

Compact crab cavity (Motivation)

- LHC needs compact cavities because of dimensional constraints. Compact cavity requires a cavity aperture of less than 150 mm which could be consistent with the beam separation of 194 mm.



- Proposed compact cavities which are compatible with LHC local option, such as parallel bar design (JLab & ODU), 400MHz ridged waveguide (SLAC), etc.
- Develop simulation model of parallel bar design of JLab & ODU, because the design fits both horizontal and vertical crabbing schemes, the deflecting mode is the lowest frequency mode, and the EM fields inside the cavity are provided.

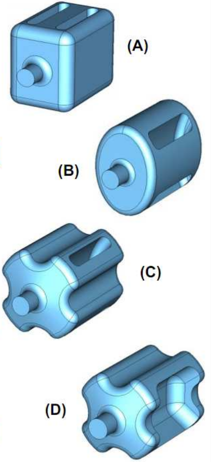
COLDEX (location for crab cavity in SPS)

- Suggested location for crab cavity.

TEM-mode crab cavity (Parallel bar design)

Cavity Properties – 400 MHz Designs

Parameter	(A)	(B)	(C)	(D)	Unit
Frequency of π mode	400.0	400.0	400.0	400.0	MHz
λ/2 of π mode	374.7	374.7	374.7	374.7	mm
Frequency of 0 mode	411.0	687.0	541.5	665.9	MHz
Nearest mode to π mode	411.0	611.6	541.5	619.6	MHz
Cavity reference length	444.7	445.0	525.0	525.0	mm
Cavity width / diameter	300.0	290.0	404.5	373.0	mm
Cavity height	383.2	408.6	404.5	373.0	mm
Bars length	330.0	330.0	330.0	330.0	mm
Bars width	55.0	60.0	60.0	-	mm
Aperture diameter	84.0	84.0	84.0	84.0	mm
Deflecting voltage (V_T^*)	0.375	0.375	0.375	0.375	MV
Peak electric field (E_P^*)	2.2	3.4	3.3	3.7	MV/m
Peak magnetic field (B_P^*)	7.9	7.71	8.2	8.3	mT
B_P^* / E_P^*	3.6	2.27	2.45	2.24	mT / (MV/m)
Geometrical factor ($G = QR_S$)	74.1	109.4	81.3	88.4	Ω
$[R/Q]_T$	413.34	255.68	372.83	285.25	Ω
$R_T R_S$	3.1×10^4	2.8×10^4	3.0×10^4	2.5×10^4	Ω^2
At $E_T^* = 1$ MV/m					



(J. Delayen, LARP CM16)

TEM-mode crab cavity (Simulation model)

- Electromagnetic fields inside the cavity are available numerically at grid points (No analytic formula).
- The electric and magnetic field of actual cavity design contains all vector components, i.e., $\mathbf{E} = E_x\hat{x} + E_y\hat{y} + E_z\hat{z}$ and $\mathbf{H} = H_x\hat{x} + H_y\hat{y} + H_z\hat{z}$.
- Eectric and magnetic fields are

$$\tilde{\mathbf{E}}(x,y,z,t) = \mathbf{E}(x,y,t)\sin\left(\omega\left(t-\frac{z}{\beta c}\right)\right),$$
$$\tilde{\mathbf{H}}(x,y,z,t) = \mathbf{H}(x,y,t)\cos\left(\omega\left(t-\frac{z}{\beta c}\right)\right).$$

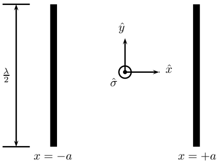
- Equation of motion becomes

$$\frac{dp_x}{dt} = \frac{q}{p_0}E_x\sin\left(\omega\left(t-\frac{z}{\beta c}\right)\right) - \frac{q\beta c}{p_0}\frac{1}{\mu_0}H_y\cos\left(\omega\left(t-\frac{z}{\beta c}\right)\right),$$
$$\frac{dp_y}{dt} = \frac{q}{p_0}E_y\sin\left(\omega\left(t-\frac{z}{\beta c}\right)\right) + \frac{q\beta c}{p_0}\frac{1}{\mu_0}H_x\cos\left(\omega\left(t-\frac{z}{\beta c}\right)\right),$$
$$\frac{dp_z}{dt} = \frac{q}{p_0}E_\sigma\sin\left(\omega\left(t-\frac{z}{\beta c}\right)\right).$$

- Using thin-lens approximation, the kicks are calculated at grid points. Interpolation scheme is applied to get the kick at any (x, y, z).

Benchmark of analytic model

- Simplied parallel wire cavity is considered to benchmark the simulation model of TEM-mode cavity.



TEM wave propagates along the y-direction.
Beam direction is along $\hat{\sigma}$.

- Electric and magnetic fields are

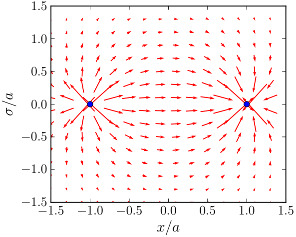
$$E_x\,(x,\sigma) = -\frac{aq}{\pi\epsilon_0}\left[\frac{x^2 - a^2 - \sigma^2}{r_-^2\, r_+^2}\right],$$

$$E_\sigma\,(x,\sigma) = -\frac{aq}{\pi\epsilon_0}\left[\frac{2x\sigma}{r_-^2\, r_+^2}\right],$$

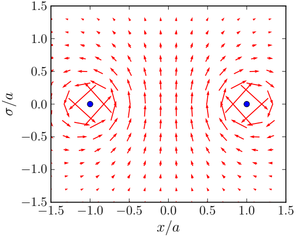
$$B_x\,(x,\sigma) = -\frac{1}{c}E_\sigma\,(x,\sigma)\,,\quad B_\sigma\,(x,\sigma) = \frac{1}{c}E_x\,(x,\sigma)\,.$$

$$r_\pm = (x \pm a)^2 + \sigma^2.$$

(Electric field)

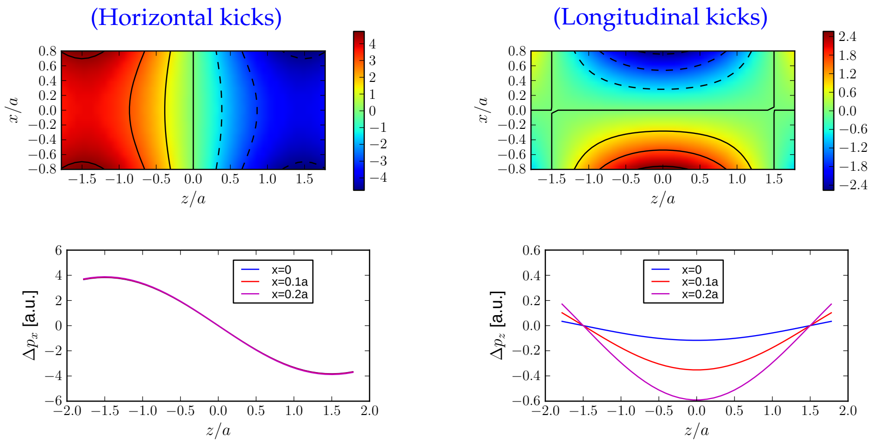


(Magnetic field)



Benchmark of analytic model

- Kicks due to the TEM-mode cavity are calculated. Vertical kicks are neglible.

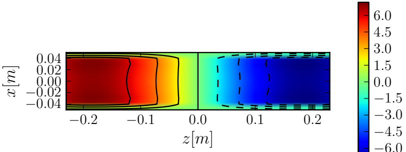


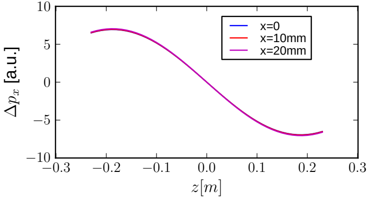
- In the model, $a = 125$ mm. At the (proposed) crab cavity location of SPS, the rms beam size is $\sigma_x = 0.91$ mm.

Cavity kicks (Parallel bar design at JLab & ODU)

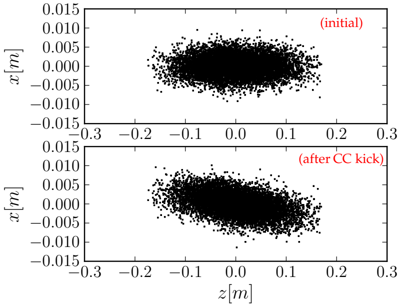
- Kicks due to the TEM-mode cavity are calculated using the electromagnetic field provided by J. Delayen.
- Actual EM fields are available only numerically at grid points. This example uses the number of grid points, $(n_x, n_y, n_z) = (11, 11, 47)$.

(Horizontal kicks)





(Deflecting angle)



Summary

- Emittance growth rate studies for nominal and low γ_T optics. Noises affect low γ_T optics (two times) less than nominal optics.
- Space-charge increases emittance in both nominal and low γ_T optics. The emittance growth is approximately 7%/hr in both horizontal and vertical planes at 55 GeV, 2.5%/hr at 120 GeV, and $\simeq$0%/hr at 270 GeV.
- Tune modulation has no effect on emittance growth at nominal tune (0.13, 0.18). However, the spectral lines in the SPS now may be different.

- Proposed crab cavity simulation model using electromagnetic fields of crab cavity from JLab & ODU.
- The model is implemented in BBSIM, and under test.

Thank you for your attention!